BRIEF COMMUNICATION

MEAN BLOOD PRESSURE AND VELOCITY FOR CALCULATION OF VENTRICULAR SYSTOLIC WORK

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Summary: In physiology, the mean arterial pressure is defined as an average pressure during one or several cardiac cycles. When calculus is not used, the mean pressure is approximately calculated as the diastolic pressure plus one third of the pulse pressure. In this article it is demonstrated that, when ventricular systolic work is concerned, the above definition of mean pressure must be replaced by a weighted average during the ejection phase of the systole. This gives a formula, by which a much higher estimate of the mean pressure is obtained.

Key words: *Heart work; Mean pressure; Mean square speed of the blood; Blood flow*

The heart ventricle physical work during the ejection phase is given by the formula

$$W = \int_0^{SV} p_{ven}\, dV$$

with SV as the stroke volume, p_{ven} as the pressure in the ventricle, and dV as an element of its volume. The formula is derived under the assumption that pressure is a function of volume only. This means that at every place of the ventricle the pressure is the same at any given time. It is evident that during the ejection phase, this assumption is not fulfilled. Also, because of the difficulty of computing heart work using this formula, the heart work is usually calculated as the energy of expelled blood to which it numerically equals. This energy consists of two parts: pressure potential energy and kinetic energy. The elementary energy (dE) imparted to the volume element of ejected blood (dV) is

$$dE = p\, dV + 0.5\, \rho \upsilon^2\, dV$$

where ρ is the blood density, υ is its velocity when leaving the ventricle, and p is the pressure at aorta or pulmonary artery root. Note that p is different from p_{ven} in the corresponding ventricle. So, the energy (E) imparted to stroke volume (SV) is

$$E = \int_0^{SV} p\, dV + 0.5\, \rho \int_0^{SV} \upsilon^2\, dV \qquad (1)$$

The ventricular systolic work is usually computed, when the mean pressure and mean square velocity of the blood are known, by a well known formula

$$E = p_{mean}\, SV + 0.5\, \rho \upsilon^2{}_{mean}\, SV \qquad (2)$$

Comparing (1) with (2), for the mean pressure it follows

$$p_{mean}\, SV = \int_0^{SV} p\, dV \;\Rightarrow\; p_{mean} = \left\{ \int_0^{SV} p\, dV \right\} / SV$$

Element of ejected blood volume $dV = Q\, dt$, where Q is the blood flow and dt is the time element. Using this, the stroke volume is given by the formula

$$SV = \int_0^{t_e} Q\, dt$$

where t_e is the ejection time. So, the final formula for the mean pressure is

$$p_{mean} = \left\{ \int_0^{t_e} p\, Q\, dt \right\} / \int_0^{t_e} Q\, dt \qquad (3)$$

From this it is seen that, when averaging over time, the product of pressure and flow must be taken into account, not only pressure. The flow (Q) is something like a weight factor.

Similarly, the formula for the mean square velocity is

$$\upsilon^2{}_{mean} = \left\{ \int_0^{t_e} \upsilon^2\, Q\, dt \right\} / \int_0^{t_e} Q\, dt$$

The mean pressure in arteries is usually calculated by the formula:

$$p_{mean} = p_{diast} + (p_{syst} - p_{diast}) / 3 \qquad (4)$$

This formula is found when averaging pressure with respect to time during the cardiac cycle (T), under the assumption that the time of diastole is twice as long as the time of systole (t_s), and pressure in diastole is p_{diast}, and in systole p_{syst}:

$$p_{mean} = \left\{ \int_0^T p \, \mathrm{d}t \right\} / T =$$

$$= \left\{ \int_0^{2t_s} p_{diast} \, \mathrm{d}t + \int_{2t_s}^{3t_s} p_{syst} \, \mathrm{d}t \right\} / (3 \, t_s) =$$

$$= \left\{ 2 \, t_s \, p_{diast} + t_s \, p_{syst} \right\} / (3 \, t_s) =$$

$$= (2 \, p_{diast} + p_{syst}) / 3 =$$

$$= p_{diast} + (p_{syst} - p_{diast}) / 3$$

The proper formula (3) would give this result if Q were constant during cardiac cycle. It is quite evident that this is not true for heart blood ejection. From this it follows that it is not correct to compute the mean pressure using (4) when ventricular systolic work is calculated.

The author compared the development in time of pressure and the blood flow during ejection given in several literary sources. Every time these functions were similar to what is seen in [1,2]. When the mean pressure is computed with formula (3), using data from figures in literature (the author, till now, could not find any other source of data), the factor *0.80* instead of *0.33* is found to be adequate:

$$p_{mean} = p_{diast} + 0{,}80 \, (p_{syst} - p_{diast}) \qquad (5)$$

When applied to physiological blood pressure values, for example, diastolic pressure *70 mm Hg* and systolic *120 mm Hg*, the mean pressure was *87 mm Hg*, while now it is *110 mm Hg*. The ratio of ventricular systolic works is approximately the same as the ratio of pressures: *110/87 = 1.26*. So, the real ventricular systolic work is approximately by one fourth greater than the ventricular systolic work when incorrectly calculated mean pressure is used.

The factor *0.80* in (5) will evidently change little when heart rate changes. Because of the causal relation between the rapid increase in blood flow and pressure increase in aorta root when ejection starts, it is expected that the pressure and blood flow functions will be similar regardless of the heart rate. If the formula (5) is valid for some pressure and blood flow functions, it will be valid also for similar functions. More exactly: Assume the pressure and blood flow functions at heart rate *HR1* are $p_{HR1}(t)$ and $Q_{HR1}(t)$ respectively and, similarly, at heart rate *HR2* thay are $p_{HR2}(t)$ and $Q_{HR2}(t)$ respectively and assume it holds

$$p_{HR1}(t) = k \, p_{HR2}(ct) \qquad (6)$$

$$Q_{HR1}(t) = K \, Q_{HR2}(ct) \qquad (7)$$

where k, K, and c are arbitrary positive constants. If the formula (5) gives the correct mean pressure for one pair of quantities, it also gives the right mean for the other pair of quantities. Proof:

$$p_{HR1,\,mean} = \left\{ \int_0^{t_e} p_{HR1}(t) \, Q_{HR1}(t) \, \mathrm{d}t \right\} / \int_0^{t_e} Q_{HR1}(t) \, \mathrm{d}t$$

$$= \left\{ \int_0^{ct_e} k \, p_{HR2}(ct) \, Q_{HR2}(ct) \, \mathrm{d}(ct) \right\} / \int_0^{ct_e} Q_{HR2}(ct) \, \mathrm{d}(ct)$$

$$= \left\{ \int_0^{u_e} k \, p_{HR2}(u) \, Q_{HR2}(u) \, \mathrm{d}u \right\} / \int_0^{u_e} Q_{HR2}(u) \, \mathrm{d}u =$$

$$= k \, p_{HR2,\,mean}$$

This is the same relation between $p_{HR1,mean}$ and $p_{HR2,mean}$ as the one calculated using (5).

When the averaging over time during cardiac cycle is used, the relations (6) and (7) are not valid, because when heart rate increases, the duration of diastole shortens relatively more than that of systole. But, in our case, averaging only during the ejection phase of systole is performed.

The ventricular systolic work can be calculated in the above described simple way (2) only when the pressure and velocity are measured at the root of aorta or pulmonary artery. The ejected blood, while flowing in a vessel, performs a work, which is changed to the kinetic energy of the blood pressed forward, to the pressure potencial energy of the blood in the vessel, to the elastic energy of the distended vessel, and also to heat. This, of course, results in the reduction of ejected blood energy.

References

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